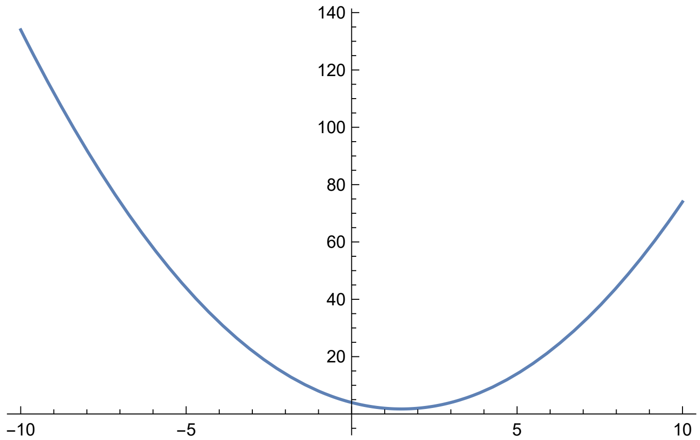
I have had Math 1200!

In[111]:=

```
Plot[x^2 - 3 x + 4, {x, -10, 10}]
```

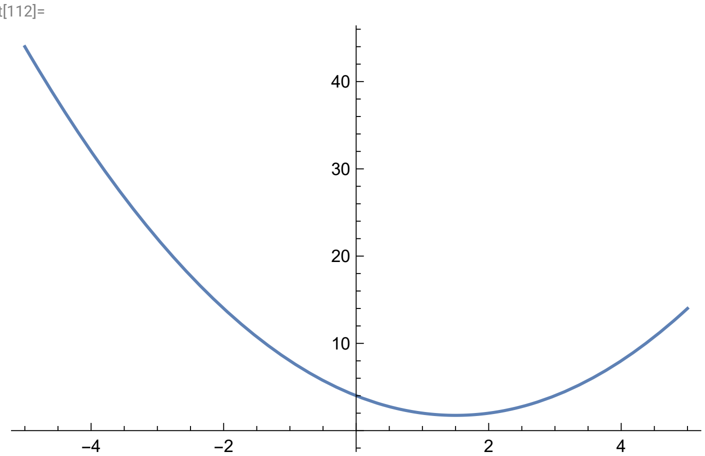
Out[111]=



In[112]:=

```
Plot[x^2 - 3 x + 4, {x, -5, 5}]
```

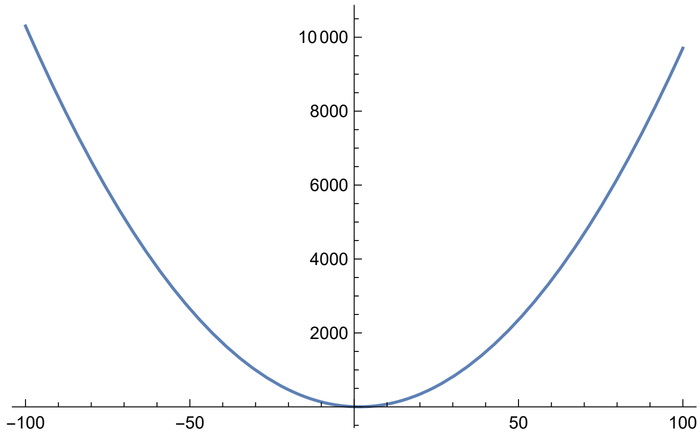
Out[112]=



In[113]:=

```
Plot[x^2 - 3 x + 4, {x, -100, 100}]
```

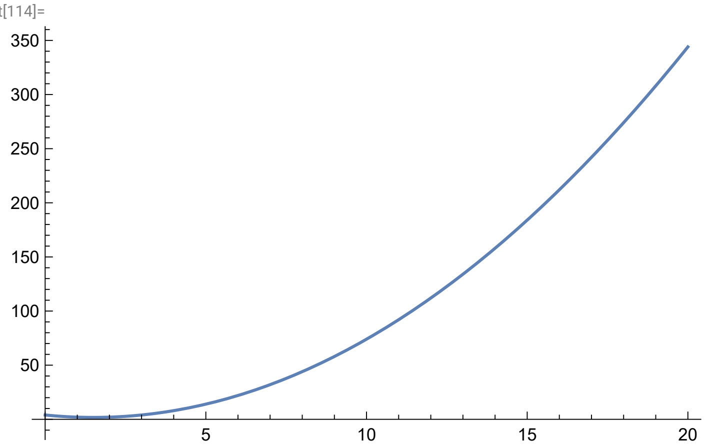
Out[113]=



In[114]:=

```
Plot[x^2 - 3 x + 4, {x, 0, 20}]
```

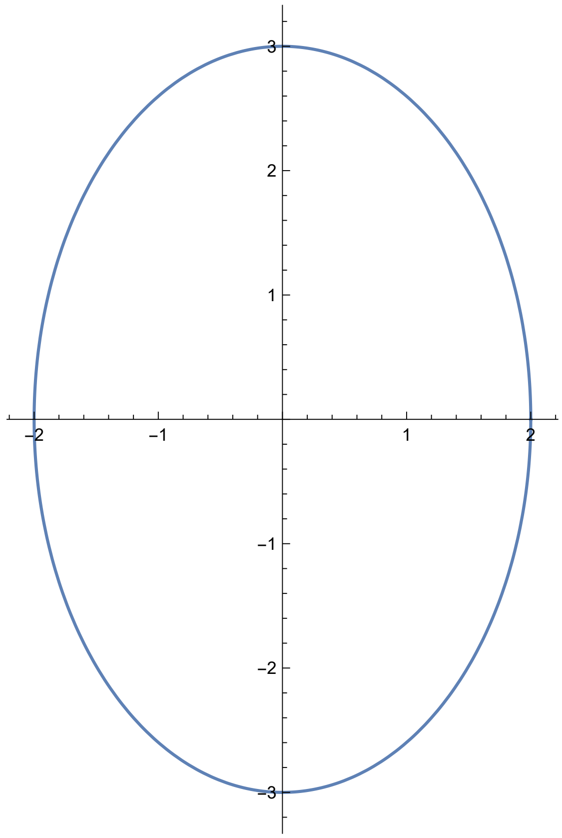
Out[114]=



In[115]:=

```
ParametricPlot[{2 Cos[t], 3 Sin[t]}, {t, 0, 2 Pi}]
```

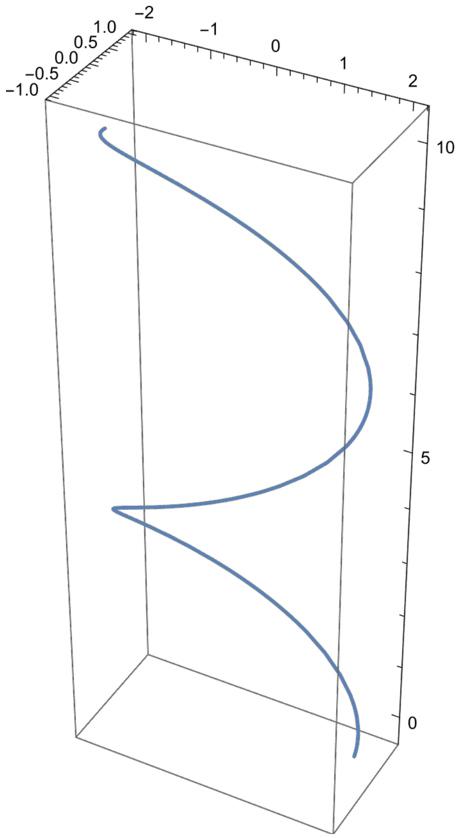
Out[115]=



In[116]:=

```
ParametricPlot3D[{2 Cos[t], Sin[t], t}, {t, 0, 10}]
```

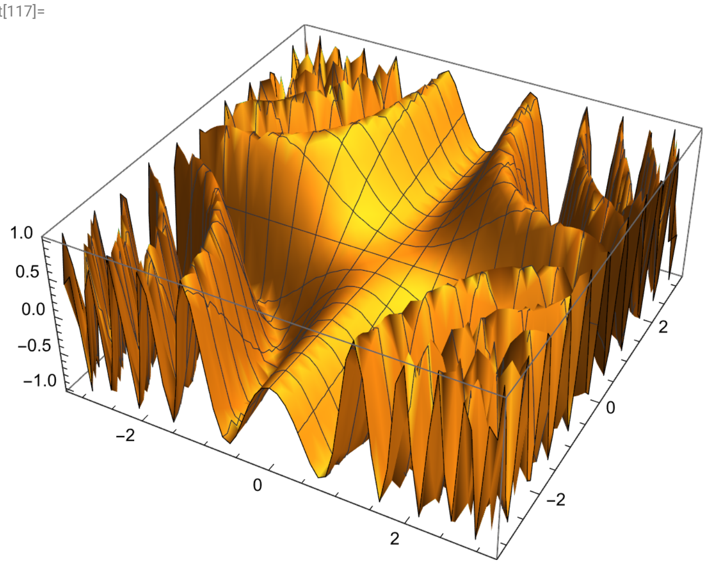
Out[116]=



In[117]:=

```
Plot3D[Sin[x^2 y], {x, -Pi, Pi}, {y, -Pi, Pi}]
```

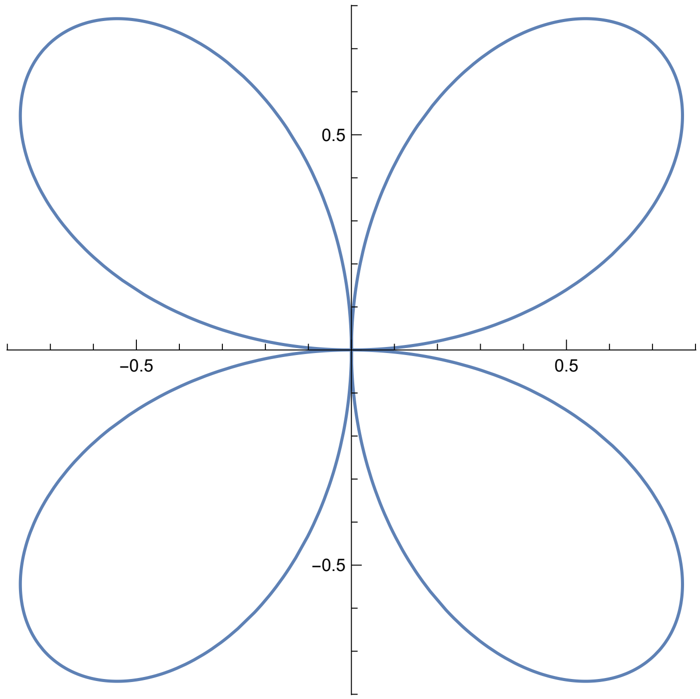
Out[117]=



In[118]:=

```
PolarPlot[Sin[2 t], {t, 0, 2 Pi}]
```

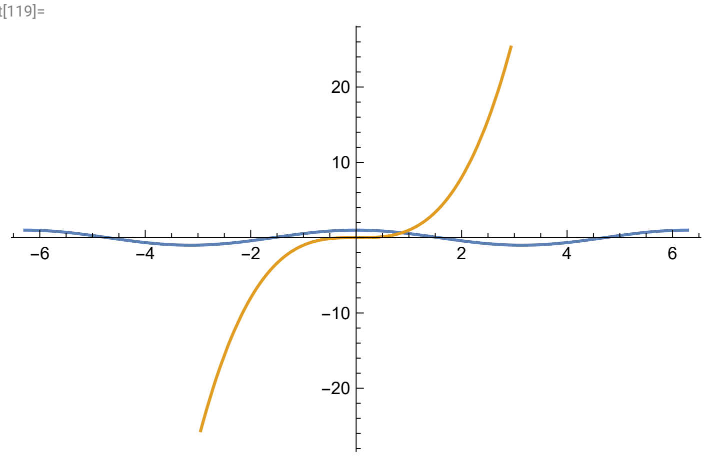
Out[118]=



In[119]:=

```
Plot[{Cos[x], x^3}, {x, -2 Pi, 2 Pi}]
```

Out[119]=



In[120]:=

```
FindRoot[Cos[x] == x^3, {x, 0.5}]
```

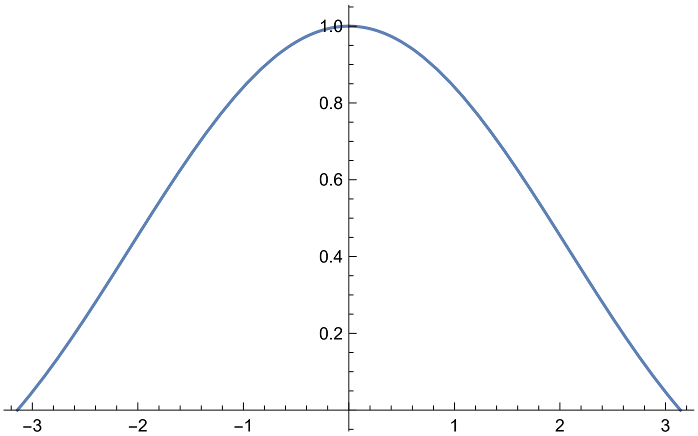
Out[120]=

```
{x → 0.865474}
```

In[121]:=

```
Plot[Sin[x] / x, {x, -Pi, Pi}]
```

Out[121]=



In[122]:=

```
Limit[Sin[x] / x, x → 0, Direction → "FromBelow"]
```

Out[122]=

1

In[123]:=

```
Limit[Sin[x] / x, x → 0, Direction → "FromAbove"]
```

Out[123]=

1

In[124]:=

```
Limit[Sin[x] / x, x → 0]
```

Out[124]=

1

Verify existence of limits!

In[125]:=

```
D[x^2 Sin[x] - 3 x + 1, x]
```

Out[125]=

$-3 + x^2 \text{Cos}[x] + 2 x \text{Sin}[x]$

In[126]:=

```
f[x_] := x^3 - 2 x^2 + 5; f'[x]
```

Out[126]=

$-4 x + 3 x^2$

In[127]:=

```
f''[x]
```

Out[127]=

$-4 + 6 x$

In[128]:=
```
D[f[x], {x, 3}]
```

Out[128]=
6

In[129]:=
```
D[x^2 y[x]^2 + x Sin[y[x]] == 1, x]
```

Out[129]=
$\text{Sin}[y[x]] + 2\,x\,y[x]^2 + x\,\text{Cos}[y[x]]\,y'[x] + 2\,x^2\,y[x]\,y'[x] == 0$

In[130]:=
```
Solve[%, y'[x]]
```

Out[130]=
$$\left\{\left\{y'[x] \to \frac{-\text{Sin}[y[x]] - 2\,x\,y[x]^2}{x\,(\text{Cos}[y[x]] + 2\,x\,y[x])}\right\}\right\}$$

In[131]:=
```
f[x_, y_, z_] := x^4 z^3 y + x Sin[z + y];
D[f[x, y, z], x]
```

Out[132]=
$4\,x^3\,y\,z^3 + \text{Sin}[y + z]$

In[133]:=
```
D[f[x, y, z], {x, 2}, {y, 1}, {z, 1}]
```

Out[133]=
$36\,x^2\,z^2$

In[134]:=
```
Integrate[x^2 + 1, x]
```

Out[134]=
$$x + \frac{x^3}{3}$$

In[135]:=
```
Integrate[x^2 + 1, {x, -1, 2}]
```

Out[135]=
6

In[136]:=
```
f[x_, y_] := 1 - (x^2 / 4) - (y^2 / 9);
Integrate[f[x, y], {x, -2, 2}, {y, -3, 3}]
```

Out[137]=
8

In[138]:=
```
Integrate[Exp[-x^2], {x, 0, Infinity}]
```

Out[138]=
$$\frac{\sqrt{\pi}}{2}$$

In[139]:=

```
Integrate[3 x^2 (2010 - x^3)^1999, x]
```

Out[139]=

3 (… 1 …)

Size in memory: 3.5 MB | Show more | Show all | Iconize | Store full expression in notebook

You can easily find the above integral using u substituion: $-\frac{1}{2000}\left(2010-x^3\right)^{2000}+C.$

In[140]:=

```
Series[E^x, {x, 0, 6}]
```

Out[140]=

$$1+x+\frac{x^2}{2}+\frac{x^3}{6}+\frac{x^4}{24}+\frac{x^5}{120}+\frac{x^6}{720}+O[x]^7$$

In[141]:=

```
Normal[Series[Cos[x], {x, 0, 8}]]
```

Out[141]=

$$1-\frac{x^2}{2}+\frac{x^4}{24}-\frac{x^6}{720}+\frac{x^8}{40320}$$

In[142]:=

```
Series[Log[x], {x, 1, 5}]
```

Out[142]=

$$(x-1)-\frac{1}{2}(x-1)^2+\frac{1}{3}(x-1)^3-\frac{1}{4}(x-1)^4+\frac{1}{5}(x-1)^5+O[x-1]^6$$

In[143]:=

```
Sum[1 / n^2, {n, 1, Infinity}]
```

Out[143]=

$$\frac{\pi^2}{6}$$

In[144]:=

```
u = {1, -3}
```

Out[144]=

{1, -3}

In[145]:=

```
MatrixForm[u]
```

Out[145]//MatrixForm=

$$\begin{pmatrix} 1 \\ -3 \end{pmatrix}$$

In[146]:=

```
v = {{-2, 5, -1}}
```

Out[146]=

{{-2, 5, -1}}

In[147]:=
```
MatrixForm[v]
```
Out[147]//MatrixForm=
$$\begin{pmatrix} -2 & 5 & -1 \end{pmatrix}$$

In[148]:=
```
m = {{1, 2}, {3, 4}, {5, 6}}
```
Out[148]=
```
{{1, 2}, {3, 4}, {5, 6}}
```
In[149]:=
```
n = {{3, 2, 1}, {6, 5, 4}}
```
Out[149]=
```
{{3, 2, 1}, {6, 5, 4}}
```
In[150]:=
```
l = {{3, -1}, {-2, 6}, {-4, 5}}
```
Out[150]=
```
{{3, -1}, {-2, 6}, {-4, 5}}
```
In[151]:=
```
m - 2 l
```
Out[151]=
```
{{-5, 4}, {7, -8}, {13, -4}}
```
In[152]:=
```
n.m
```
Out[152]=
```
{{14, 20}, {41, 56}}
```
In[153]:=
```
{{14, 20}, {41, 56}}
```
Out[153]=
```
{{14, 20}, {41, 56}}
```
In[154]:=
```
MatrixForm[n.m]
```
Out[154]//MatrixForm=
$$\begin{pmatrix} 14 & 20 \\ 41 & 56 \end{pmatrix}$$

In[155]:=
```
Dimensions[m]
```
Out[155]=
```
{3, 2}
```
In[156]:=
```
i = IdentityMatrix[3]
```
Out[156]=
```
{{1, 0, 0}, {0, 1, 0}, {0, 0, 1}}
```

In[157]:=

```
d = DiagonalMatrix[{2, 2, -3}]
```

Out[157]=

```
{{2, 0, 0}, {0, 2, 0}, {0, 0, -3}}
```

In[158]:=

```
Transpose[m]
```

Out[158]=

```
{{1, 3, 5}, {2, 4, 6}}
```

In[159]:=

```
MatrixForm[Transpose[m]]
```

Out[159]//MatrixForm=

$$\begin{pmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{pmatrix}$$

In[160]:=

```
a = {{2., 3, -5}, {-1, 4, 2}, {5, 7, 2}}
```

Out[160]=

```
{{2., 3, -5}, {-1, 4, 2}, {5, 7, 2}}
```

In[161]:=

```
Det[a]
```

Out[161]=

```
159.
```

In[162]:=

```
Inverse[a]
```

Out[162]=

```
{{-0.0377358, -0.257862, 0.163522},
 {0.0754717, 0.18239, 0.00628931}, {-0.169811, 0.00628931, 0.0691824}}
```

In[163]:=

```
MatrixForm[Inverse[a]]
```

Out[163]//MatrixForm=

$$\begin{pmatrix} -0.0377358 & -0.257862 & 0.163522 \\ 0.0754717 & 0.18239 & 0.00628931 \\ -0.169811 & 0.00628931 & 0.0691824 \end{pmatrix}$$

In[164]:=

```
MatrixPower[a, 3]
```

Out[164]=

```
{{-101., -238., 58.}, {66., 239., 68.}, {-66., 218., 35.}}
```

In[165]:=

```
Eigenvalues[a]
```

Out[165]=

```
{6.52411 + 0. i, 0.737944 + 4.88125 i, 0.737944 - 4.88125 i}
```

In[166]:=

```
Eigenvectors[a]
```

Out[166]=

```
{{-0.312408 + 0. i, 0.662901 + 0. i, 0.680414 + 0. i},
 {0.746572 + 0. i, -0.12497 + 0.189974 i, 0.113461 - 0.614857 i},
 {0.746572 + 0. i, -0.12497 - 0.189974 i, 0.113461 + 0.614857 i}}
```

In[167]:=

```
Eigensystem[a]
```

Out[167]=

```
{{6.52411, 0.737944 + 4.88125 i, 0.737944 - 4.88125 i},
 {{-0.312408 + 0. i, 0.662901 + 0. i, 0.680414 + 0. i},
  {0.746572 + 0. i, -0.12497 + 0.189974 i, 0.113461 - 0.614857 i},
  {0.746572 + 0. i, -0.12497 - 0.189974 i, 0.113461 + 0.614857 i}}}
```

In[168]:=

```
Solve[{2 x + 3 y - 5 z == 14, -x + 4 y + 2 z == 0, 5 x + 7 y + 2 z == -9}, {x, y, z}]
```

Out[168]=

```
{{x → -2, y → 1, z → -3}}
```

In[169]:=

```
b = {14, 0, -9}
```

Out[169]=

```
{14, 0, -9}
```

In[170]:=

```
Inverse[a].b
```

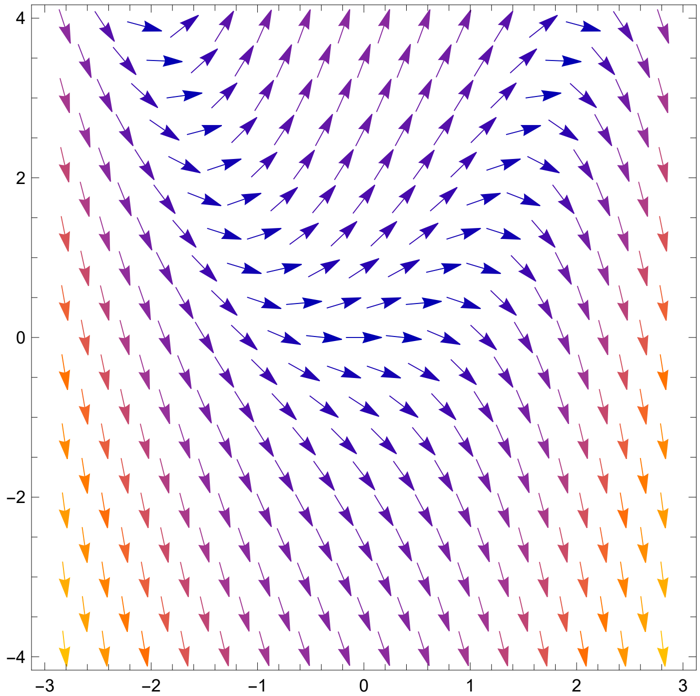
Out[170]=

```
{-2., 1., -3.}
```

In[171]:=

```
VectorPlot[{1, y - x^2}, {x, -3, 3}, {y, -4, 4}]
```

Out[171]=

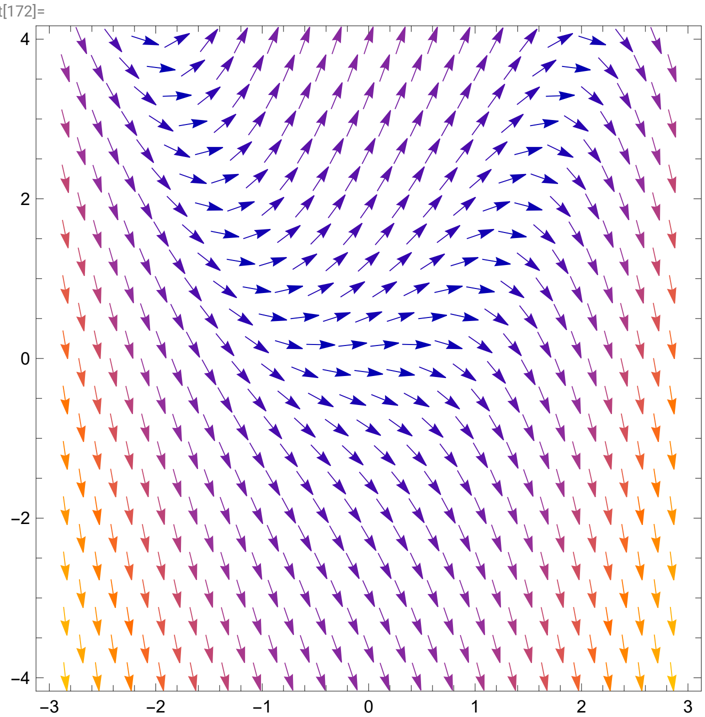


The above is not the direction field of the ODE since all vectors have a same length. The color indicates the relative length of vectors.

In[172]:=

```
VectorPlot[{1, y - x^2}, {x, -3, 3}, {y, -4, 4}, VectorPoints → 20]
```

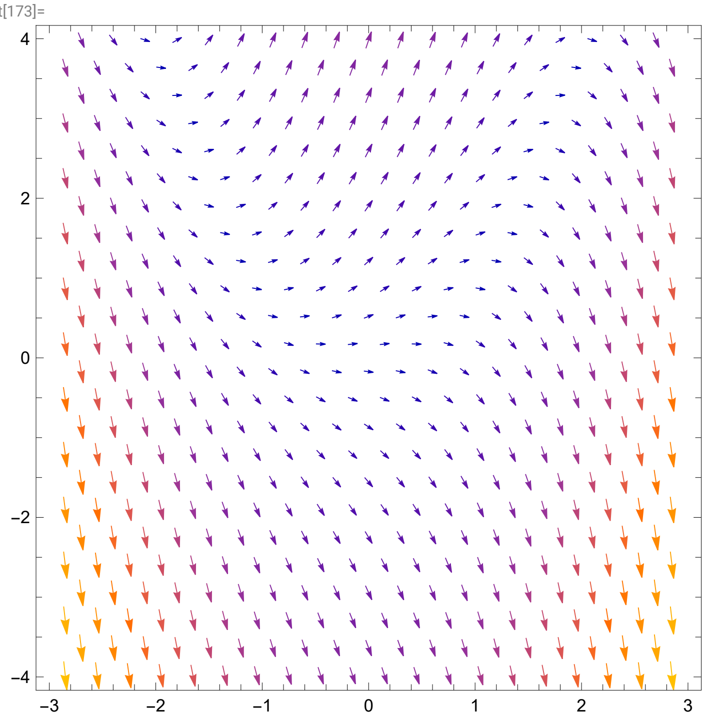
Out[172]=



In[173]:=

```
VectorPlot[{1, y - x^2}, {x, -3, 3}, {y, -4, 4}, VectorScaling → Automatic, VectorPoints → 20]
```

Out[173]=

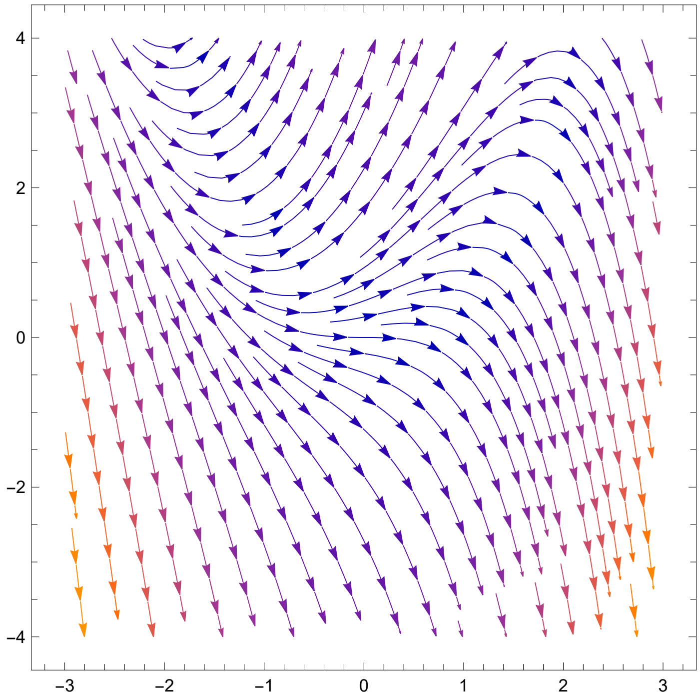


The above is the direction field of the ODE $\frac{dy}{dx} = y - x^2$.

In[174]:=

```
StreamPlot[{1, y - x^2}, {x, -3, 3}, {y, -4, 4}]
```

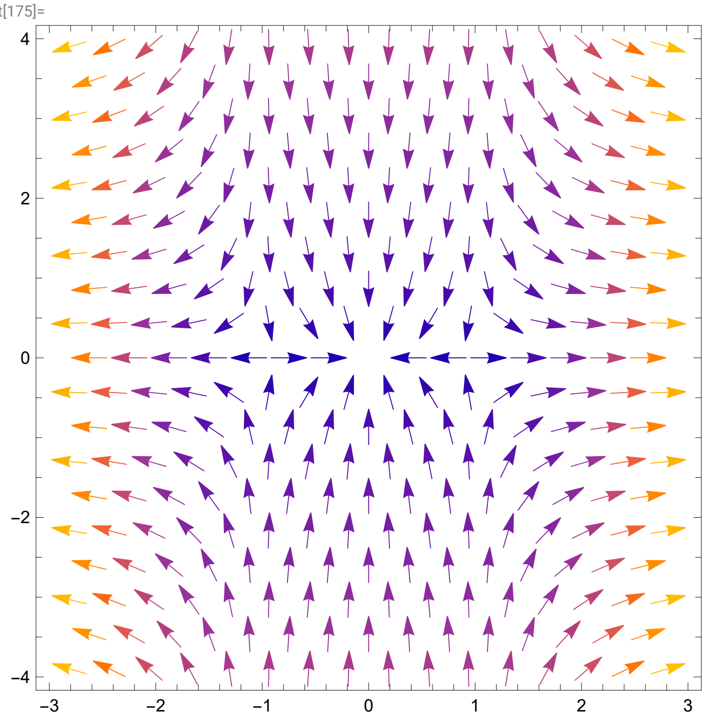
Out[174]=



In[175]:=

```
VectorPlot[{-x + x^3, -2 y}, {x, -3, 3}, {y, -4, 4}]
```

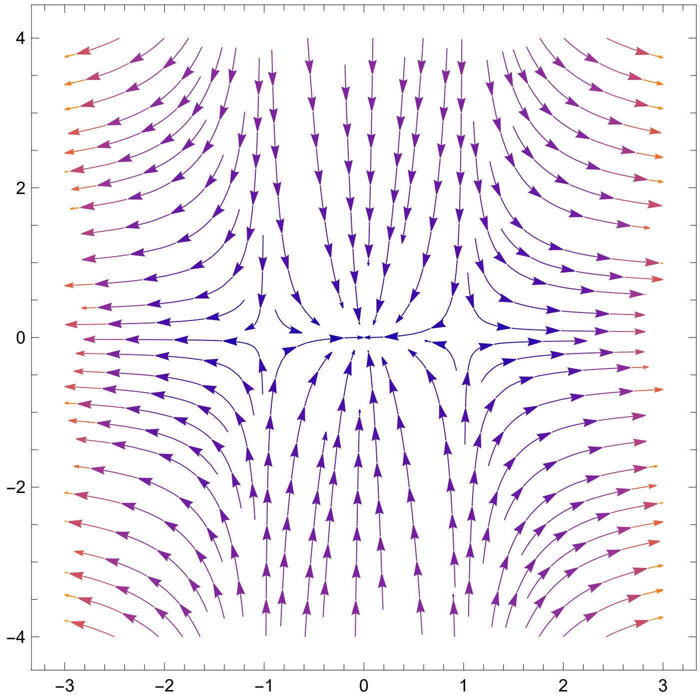
Out[175]=



In[176]:=

```
StreamPlot[{-x + x^3, -2 y}, {x, -3, 3}, {y, -4, 4}]
```

Out[176]=



In[177]:=

```
DSolve[y'[x] == y[x] - x^2, y[x], x]
```

Out[177]=

$$\{\{y[x] \to 2 + 2x + x^2 + e^x c_1\}\}$$

The general solution of the above ODE is $y = Ce^x + x^2 + 2x + 2$.

In[178]:=

```
DSolve[{y'[x] == y[x] - x^2, y[0] == 2}, y[x], x]
```

Out[178]=

$$\{\{y[x] \to 2 + 2x + x^2\}\}$$

In[179]:=

```
DSolve[{x'[t] == 2 x[t] + y[t], y'[t] == x[t] - 2 y[t]}, {x[t], y[t]}, t]
```

Out[179]=

$$\left\{\left\{x[t] \to \frac{1}{10} e^{-\sqrt{5}t} \left(5 - 2\sqrt{5} + 5 e^{2\sqrt{5}t} + 2\sqrt{5} e^{2\sqrt{5}t}\right) c_1 + \frac{e^{-\sqrt{5}t}\left(-1 + e^{2\sqrt{5}t}\right) c_2}{2\sqrt{5}},\right.\right.$$

$$\left.\left. y[t] \to \frac{e^{-\sqrt{5}t}\left(-1 + e^{2\sqrt{5}t}\right) c_1}{2\sqrt{5}} - \frac{1}{10} e^{-\sqrt{5}t} \left(-5 - 2\sqrt{5} - 5 e^{2\sqrt{5}t} + 2\sqrt{5} e^{2\sqrt{5}t}\right) c_2\right\}\right\}$$

In[180]:=

```
DSolve[{x'[t] == 2 x[t] + y[t], y'[t] == x[t] - 2 y[t], x[0] == 2, y[0] == 1}, {x[t], y[t]}, t]
```

Out[180]=

$$\left\{\left\{x[t] \to \frac{1}{2} e^{-\sqrt{5}\, t} \left(2 - \sqrt{5} + 2 e^{2 \sqrt{5}\, t} + \sqrt{5}\, e^{2 \sqrt{5}\, t}\right),\ y[t] \to \frac{1}{2} e^{-\sqrt{5}\, t} \left(1 + e^{2 \sqrt{5}\, t}\right)\right\}\right\}$$

In[181]:=

```
NDSolve[{y'[x] == y[x] - x^2, y[0] == 2}, y[x], {x, -3, 3}]
```

Out[181]=

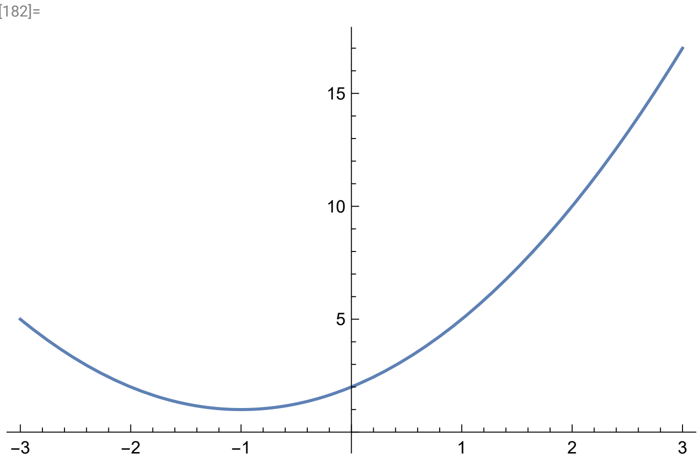
{{y[x] → InterpolatingFunction[ Domain: {{-3., 3.}} Output: scalar][x]}}

The numerical solution can be graphed, as shown below.

In[182]:=

```
Plot[Evaluate[y[x] /. %], {x, -3, 3}]
```

Out[182]=



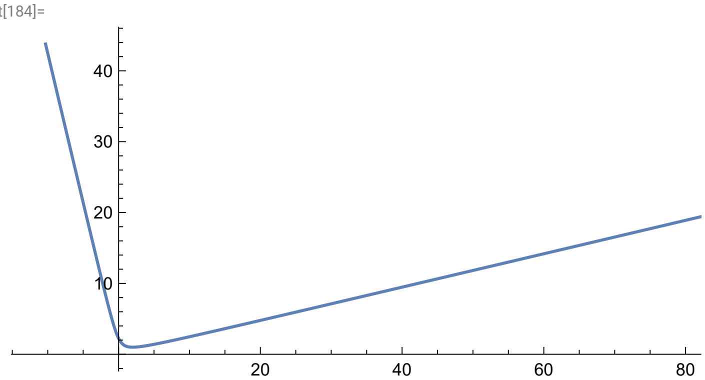
In[183]:=

```
NDSolve[{x'[t] == 2 x[t] + y[t], y'[t] == x[t] - 2 y[t], x[0] == 2, y[0] == 1},
 {x[t], y[t]}, {t, -2, 2}]
ParametricPlot[Evaluate[{x[t], y[t]} /. %], {t, -2, 2}]
```

Out[183]=

```
{{x[t] → InterpolatingFunction[ Domain: {{-2., 2.}} Output: scalar ][t],
  y[t] → InterpolatingFunction[ Domain: {{-2., 2.}} Output: scalar ][t]}}
```

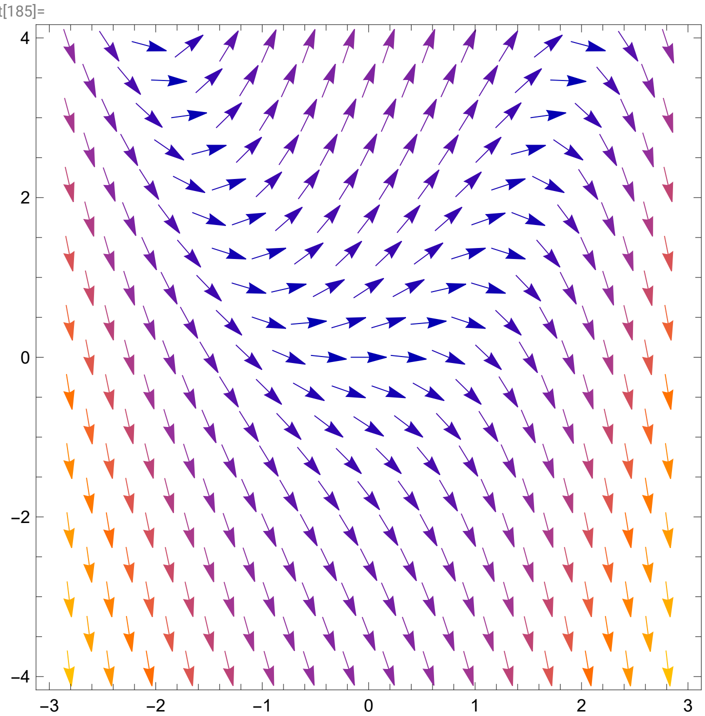
Out[184]=



In[185]:=

```
graph1 = VectorPlot[{1, y - x^2}, {x, -3, 3}, {y, -4, 4}]
```

Out[185]=



In[186]:=

```
NDSolve[{y'[x] == y[x] - x^2, y[0] == 2}, y[x], {x, -3, 3}]
```

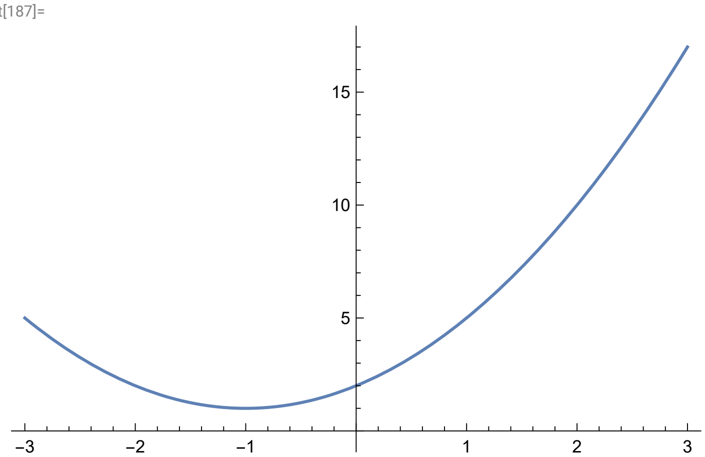
Out[186]=

```
{{y[x] → InterpolatingFunction[ Domain: {{-3., 3.}} Output: scalar ][x]}}
```

In[187]:=

```
graph2 = Plot[Evaluate[y[x] /. %], {x, -3, 3}]
```

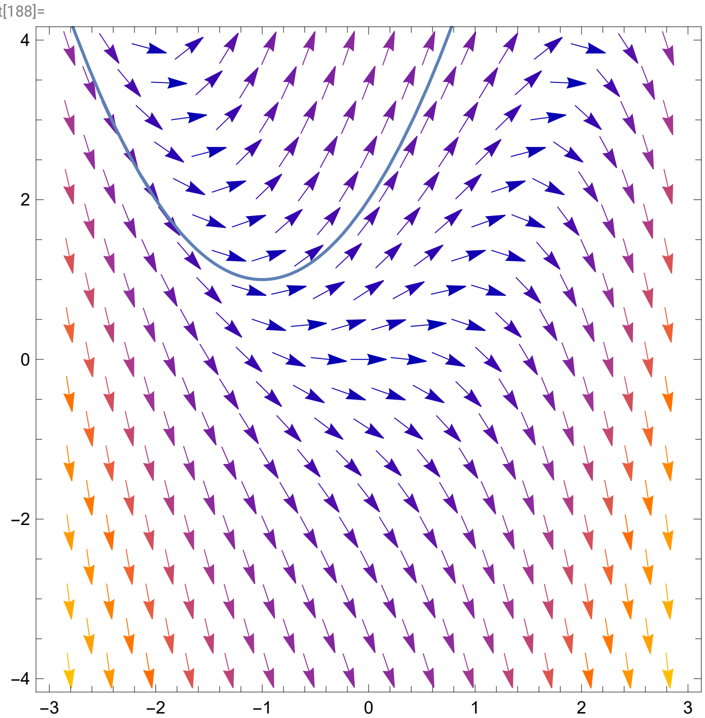
Out[187]=



In[188]:=

```
Show[graph1, graph2]
```

Out[188]=

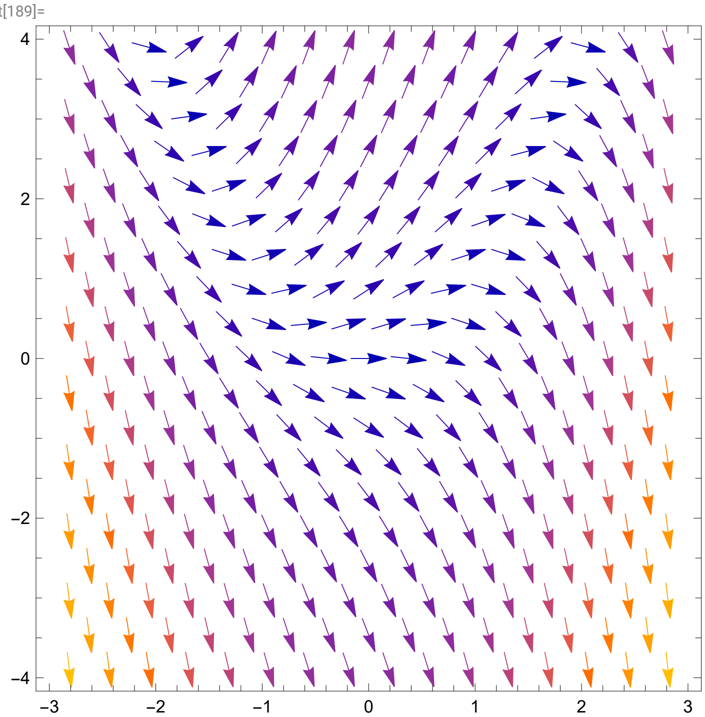


In the above the vector field and one solution are shown together.

In[189]:=

```
graph1 = VectorPlot[{1, y - x^2}, {x, -3, 3}, {y, -4, 4}]
```

Out[189]=



In[190]:=

```
Table[NDSolve[{y'[x] == y[x] - x^2, y[0] == n}, y[x], {x, -3, 3}], {n, -3, 3, 1}]
```

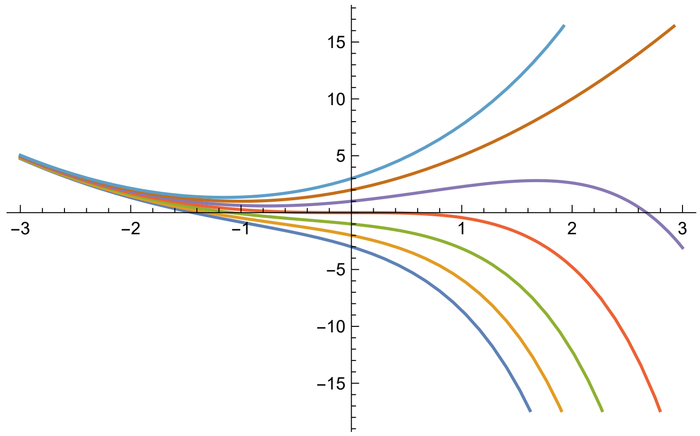
Out[190]=

```
{{{y[x] → InterpolatingFunction[ Domain: {{-3., 3.}} Output: scalar ][x]}},
 {{y[x] → InterpolatingFunction[ Domain: {{-3., 3.}} Output: scalar ][x]}},
 {{y[x] → InterpolatingFunction[ Domain: {{-3., 3.}} Output: scalar ][x]}},
 {{y[x] → InterpolatingFunction[ Domain: {{-3., 3.}} Output: scalar ][x]}},
 {{y[x] → InterpolatingFunction[ Domain: {{-3., 3.}} Output: scalar ][x]}},
 {{y[x] → InterpolatingFunction[ Domain: {{-3., 3.}} Output: scalar ][x]}},
 {{y[x] → InterpolatingFunction[ Domain: {{-3., 3.}} Output: scalar ][x]}}}
```

In[191]:=

```
graph3 = Plot[Evaluate[y[x] /. %], {x, -3, 3}]
```

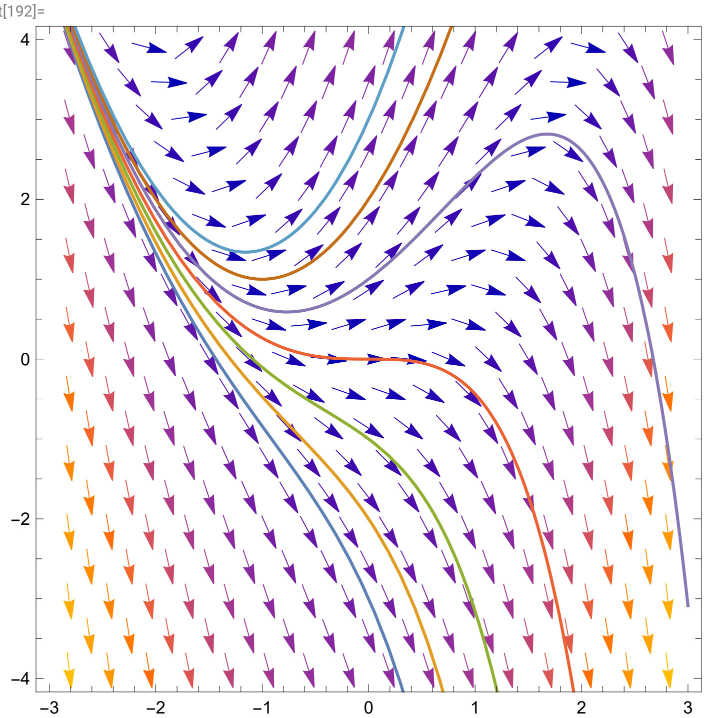
Out[191]=



In[192]:=

```
Show[graph1, graph3]
```

Out[192]=



In the above the vector field and several solutions are shown together.

Above, we have defined matrix a and set a value for n, so we need to clear those before reusing a and n.

In[193]:=

```
Clear[a, n]
RecurrenceTable[{a[n + 1] == 0.5 a[n] + 1, a[0] == 3}, a, {n, 0, 5}]
```

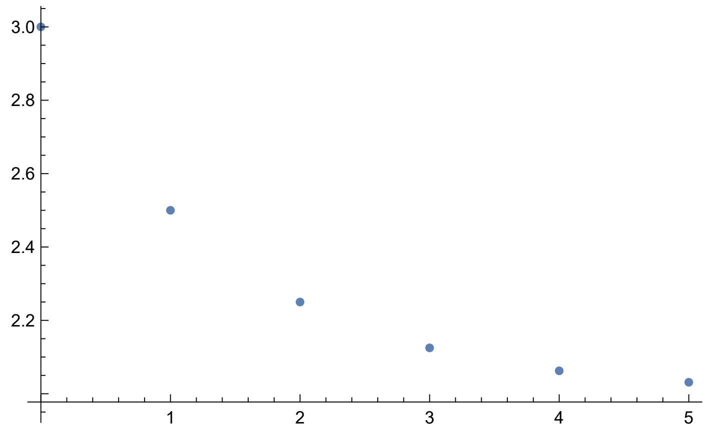
Out[194]=

```
{3., 2.5, 2.25, 2.125, 2.0625, 2.03125}
```

In[195]:=

```
a[0] = 3;
a[n_] := 0.5 (a[n - 1]) + 1
ListPlot[Table[{n, a[n]}, {n, 0, 5}]]
```

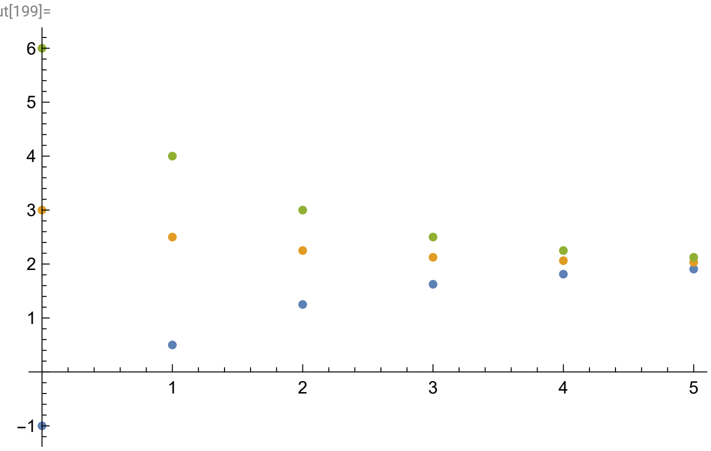
Out[197]=



In[198]:=

```
a[n_] := 0.5 a[n - 1] + 1
ListPlot[Table[Table[{n, a[n]}, {n, 0, 5}], {a[0], {-1, 3, 6}}]]
```

Out[199]=



In[200]:=

```
Clear[a]
RSolve[{a[n + 1] == 0.5 a[n] + 1, a[0] == 3}, a[n], n]
```

Out[201]=

$\{\{a[n] \to 2.^{-1.\,n}\,(1. + 2.^{1+n})\}\}$

We have also defined b above, so we need to clear it before reusing b.

In[202]:=

```
Clear[b]
RecurrenceTable[{a[n + 1] == a[n] (1.1 - 0.01 b[n]) + 1,
  b[n + 1] == b[n] (-0.1 + 0.05 a[n]), a[0] == 30, b[0] == 5}, {a, b}, {n, 0, 5}]
```

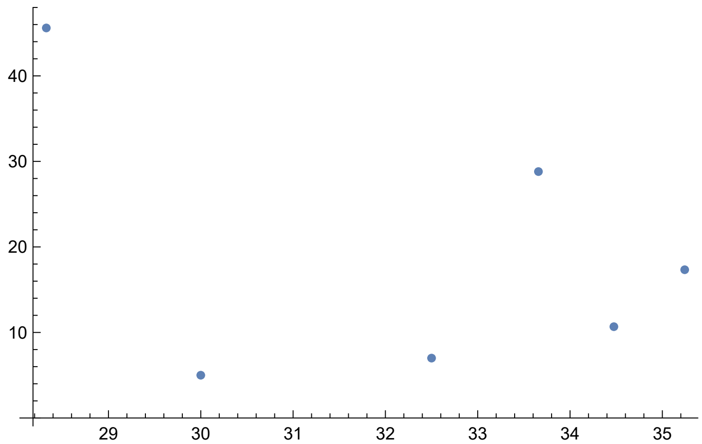
Out[203]=

```
{{30., 5.}, {32.5, 7.}, {34.475, 10.675},
 {35.2423, 17.3335}, {33.6578, 28.8103}, {28.3267, 45.6035}}
```

In[204]:=

```
a[0] = 30; b[0] = 5;
a[n_] := a[n - 1] (1.1 - 0.01 b[n - 1]) + 1; b[n_] := b[n - 1] (-0.1 + 0.05 a[n - 1])
ListPlot[Table[{a[n], b[n]}, {n, 0, 5}]]
```

Out[206]=



In[207]:=

```
Clear[a, b]
RSolve[{a[n] == b[n - 1] + n, b[n] == a[n - 1] - n, a[1] == b[1] == 1}, {a[n], b[n]}, n]
```

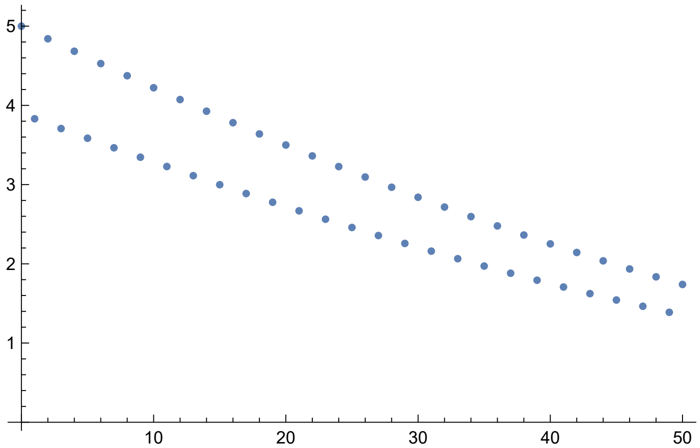
Out[208]=

$$\left\{\left\{a[n] \to \frac{1}{4}\left(4 + 3(-1)^n + (-1)^{2n} + 2(-1)^{2n} n\right),\ b[n] \to \frac{1}{4}\left(4 - 3(-1)^n - (-1)^{2n} - 2(-1)^{2n} n\right)\right\}\right\}$$

In[209]:=

```
xvalues = Table[n, {n, 0, 50}];
yvalues =
  RecurrenceTable[{a[n] == a[n - 1]^1.01 + 0.25 (-1)^n a[n - 1], a[0] == 5}, a, {n, 0, 50}];
points = Transpose[{xvalues, yvalues}];
ListPlot[points]
```

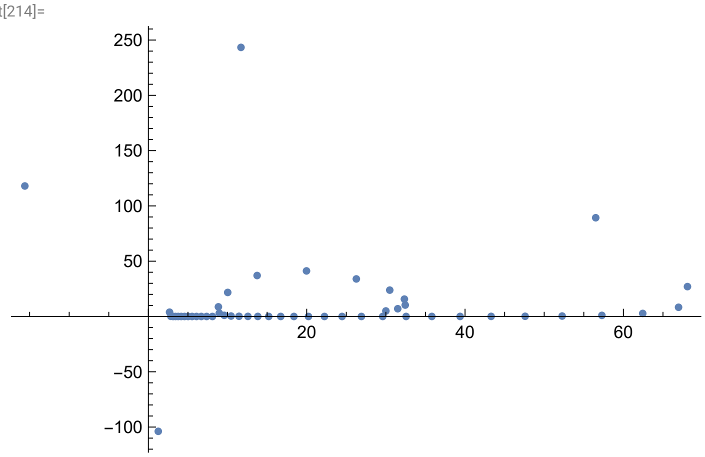
Out[212]=



In[213]:=

```
points = RecurrenceTable[{a[n + 1] == a[n] (1.1 - 0.01 b[n]),
    b[n + 1] == b[n] (-0.1 + 0.05 a[n]), a[0] == 30, b[0] == 5}, {a, b}, {n, 0, 50}];
ListPlot[points, PlotRange → All]
```

Out[214]=



In[215]:=

```
points = RecurrenceTable[{a[n + 1] == a[n] (1.1 - 0.01 b[n]),
    b[n + 1] == b[n] (-0.1 + 0.05 a[n]), a[0] == 30, b[0] == 5}, {a, b}, {n, 0, 50}];
apoints = Table[{i - 1, points[[i, 1]]}, {i, 1, 51}];
bpoints = Table[{i - 1, points[[i, 2]]}, {i, 1, 51}];
ListPlot[{apoints, bpoints},
 PlotStyle → {{Red, PointSize[0.02]}, {Blue, PointSize[0.01]}}, PlotRange → All]
```

Out[217]=